HOW TO APPROACH QUANTUM GRAVITY – BACKGROUND INDEPENDENCE IN 1+1 DIMENSIONS

D. Grumiller[*] and W. Kummer[†]

Institute f. Theor. Physics

Vienna University of Technology

Wiedner Hauptstr. 8-10, A-1040 Vienna, Austria

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Abstract

The application of quantum theory to gravity is beset with many technical and conceptual problems. After a short tour d'horizon of recent attempts to master those problems by the introduction of new approaches, we show that the aim, a background independent quantum theory of gravity, can be reached in a particular area, 2d dilaton quantum gravity, *without* any assumptions beyond standard quantum field theory.

1 Introduction

It has been realized for some time that a merging of quantum theory with Einstein's theory of general relativity[1] (GR) is necessitated by consistency arguments. In *Gedankenexperimenten* the interaction of a classical gravitational wave with a quantum system inevitably leads to contradictions [3]. Arguments of this type are important because no relevant experimental data are available – we are very far from the quantum gravity analogue of the Balmer series.

On the other hand, when a quantum theory (QT) of gravity is developed along usual lines, one is confronted with a fundamental problem, from which many other (secondary) difficulties can be traced. The crucial difference to quantum field theory (QFT) in flat space is the fact that the variables of gravity exhibit a dual role, they are fields living on a manifold which is

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[*]grumil@hep.itp.tuwien.ac.at

[†]wkummer@tph.tuwien.ac.at

[1]Several reviews on quantum gravity have emerged at the turn of the millennium, cf. e.g. [1, 2].

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determined by themselves, "stage" and "actors" coincide. But there exist also numerous other problems: the time variable, an object with special properties already in QT, in GR appears on an equal footing with the space coordinates ("problem of time" which manifests itself in many disguises); the information paradox [4]; perturbative non-renormalizability [5] etc.

In section 2 we discuss some key-points regarding the definition of physical observables in QFT and the ensuing ones in quantum gravity (QGR). Then we critically mention some "old" and "new" approaches to QGR (section 3) from a strictly quantum field theorist's point of view. Finally we give some highlights on the "Vienna approach" to 2d dilaton quantum gravity with matter, including a new result (within that approach) on entropy corrections which is in agreement with the one found in literature (section 4). In that area which contains also models with physical relevance (e.g. spherically reduced gravity) the application of just the usual concepts of (even nonpertubative!) QFT lead to very interesting consequences [6] which allow physical interpretations in terms of "solid" traditional QFT observables.

2 Observables

2.1 Cartan variables in GR

Physical observables in the sense used here are certain functionals of the field variables which are directly accessible to experimental measurements.

The metric g in GR can be considered as a "derived" field variable

$$g = e^a \otimes e^b \eta_{ab}, \tag{1}$$

because it is the direct product of the dual basis one forms[2] $e^a = e^a_\mu \, dx^\mu$ contracted with the flat local Lorentz metric η_{ab} which is used to raise and lower "flat indices" denoted by Latin letters ($\eta = \text{diag}(1, -1, -1, -1, ...)$, $x^\mu = \{x^0, x^i\}$). Local Lorentz invariance leads to the "covariant derivative" $D^a{}_b = \delta^a_b d + \omega^a{}_b$ with a spin connection 1-form $\omega^a{}_b$ as a gauge field. Its antisymmetry $\omega^{ab} = -\omega^{ba}$ implies metricity. Thanks to the Bianchi identities all covariant tensors relevant for constructing actions in even dimensions can be expressed in terms of e^a, the curvature 2-form $R^{ab} = (D\omega)^{ab}$ and the torsion 2-form $T^a = (De)^a$. For nonvanishing torsion the affine connection $\Gamma_{\mu\nu}{}^\rho = E^\rho_a \, (D_\mu e)^a_\nu$, expressed in terms of components e^a_μ and of its inverse E^ρ_a, besides the usual Christoffel symbols also contains a contorsion term in $\Gamma_{(\mu\nu)}{}^\rho$, whereas $\Gamma_{[\mu\nu]}{}^\rho$ are the components of torsion. Einstein gravity in d=4 dimensions postulates vanishing torsion $T^a = 0$ so that $\omega = \omega(e)$. This theory can be derived from the Hilbert action (G_N is Newton's constant; dS

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[2]For details on gravity in the Cartan formulation we refer to the mathematical literature, e.g. [7]

space results for nonvanishing cosmological constant Λ from the replacement $R^{ab} \rightarrow R^{ab} - \frac{4}{3}\Lambda e^a \wedge e^b$)

$$L_{(H)} = \frac{1}{16\pi G_n} \int_{\mathcal{M}_4} R^{ab} \wedge e^c \wedge e^d \epsilon_{abcd} + L_{(\text{matter})}. \tag{2}$$

Because of the "Palatini mystery", independent variation of $\delta\omega$ yields $T^a = 0$, whereas δe produces the Einstein equations.

Instead of working with the metric (1) the "new" approaches [8] are based upon a gauge field related to ω^{ab}

$$A^{ab} = \frac{1}{2}\left(\omega^{ab} - \frac{\gamma}{2}\epsilon^{ab}{}_{cd}\,\omega^{cd}\right). \tag{3}$$

The Barbero-Immirzi parameter γ [9] is an arbitrary constant. The extension to complex gravity $(\gamma = i)$ makes A^a a self-adjoint field and transforms the Einstein theory into the one of an $SU(2)$ gauge field

$$A_i^{\underline{a}} = \epsilon^{0\,\underline{a}}{}_{\underline{b}\,\underline{c}}\,A_i^{\underline{b}\,\underline{c}}, \tag{4}$$

where the index $\underline{a} = 1, 2, 3$. This formulation is the basis of loop quantum gravity and spin foam models (see below).

2.2 Observables in classical GR

The exploration of the global properties of a certain solution of (2), its singularity structure etc., is only possible by means of the introduction of an additional test field, most simply a test particle with action

$$
\begin{aligned}
L_{(\text{test})} &= -m_0 \int |ds|, \\
ds^2 &= g_{\mu\nu}\left(x\left(\tau\right)\right)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau},
\end{aligned}
\tag{5}
$$

which is another way to incorporate Einstein's old proposal [10] of a "net of geodesics". The path $x^\mu\left(\tau\right)$ is parameterized by the affine parameter τ (actually only timelike or lightlike $ds^2 \geq 0$ describes the paths of a physical particle).

It is not appreciated always that the global properties of a manifold are *defined* in terms of a specific device like (5). Whereas the usual geodesics derived from (5) depend on $g_{\mu\nu}$ through the Christoffel symbols only, e.g. in the case of torsion also the contorsion may contribute ("autoparallels") in the affine connection; spinning particles "feel" the gravimagnetic effect etc. As a consequence, when a field dependent transformation of the gravity variables is performed (e.g. a conformal transformation from a "Jordan frame" to an "Einstein frame" in Jordan-Brans-Dicke [11] theory) the action of the device must be transformed in the same way.

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2.3 Observables in QFT

In flat QFT one starts from a Schrödinger equation, dependent on field operators and, proceeding through Hamiltonian quantization to the path integral, the experimentally accessible observables are the elements of the S-matrix, or quantities expressible by those.[3] It should be recalled that the properly defined renormalized S-matrix element obtains by amputation of external propagators in the related Green function, multiplication with polarizations and with a square root of the wave function renormalization constant, taking the mass-shell limit.

In gauge theories one encounters the additional problem of gauge-dependence, i.e. the dependence on some gauge parameter β introduced by generic gauge fixing. Clearly the S-matrix elements must be and indeed are [13] independent of β. But other objects, in particular matrix-elements of gauge invariant operators $\mathcal{O}_A$, depend on β. In addition, under renormalization they mix with operators $\tilde{\mathcal{O}}_{\tilde{A}}$ of the same "twist" (dimension minus spin) which depend on Faddeev-Popov ghosts [14] and are not gauge-invariant:

$$\mathcal{O}_A^{(ren)} = Z_{AB}\mathcal{O}_B + Z_{A\tilde{B}}\tilde{\mathcal{O}}_{\tilde{B}}$$
$$\tilde{\mathcal{O}}_{\tilde{A}}^{(ren)} = \qquad\qquad Z_{\tilde{A}\tilde{B}}\tilde{\mathcal{O}}_{\tilde{B}} \qquad\qquad (6)$$

The contribution of such operators to the S-matrix element (sic!) of e.g. the scaling limit for deep inelastic scattering [15] of leptons on protons [16] occurs only through the anomalous dimensions ($\propto \partial Z_{AB}/\partial\Lambda$ for a regularisation cut-off Λ). And those objects, also thanks to the triangular form of (6), are gauge-independent!

In flat QFT, as well as in QGR, the (gauge invariant) "Wilson loop"

$$W_{(\mathcal{C})} = \operatorname{Tr} P \exp\left(i \oint_{\mathcal{C}} A_{\mu\,dx^\mu}\right), \qquad\qquad (7)$$

parameterized by a path ordered closed curve $\mathcal{C}$, often is assumed to play an important role. In covariant gauges it is multiplicatively renormalizable with the renormalization constant depending on the length of $\mathcal{C}$, the UV cut-off and eventual cusp-angles in $\mathcal{C}$ [17]. Still the relation to experimentally observable quantities (should one simply drop the renormalization constant or proceed [13] as for an S-matrix?) is unclear. Worse, for lightlike axial gauges $(nA) = 0$ $(n^2 = 0)$ multiplicative renormalization is not applicable [18]. Then, only for a matrix element of (7) between "on-shell gluons", this type of renormalization is restored. Still the renormalization constant is different from covariant gauge, except for the anomalous dimension derived from it (cf. precisely that feature of operators in deep inelastic scattering).

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[3]Note that ordinary quantum mechanics and its Schrödinger equation appear as the nonrelativistic, weak coupling limit of the Bethe-Salpeter equation of QFT [12]. Useful notions like eigenvalues of Hermitian operators, collapse of wave functions etc. are not basic concepts in this more general frame (cf. footnote 2 in ref. [13]).

3   Approaches to QGR

"Old" QGR worked with a separation of the two aspects of gravity variables by the decomposition of the metric

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}, \tag{8}$$

which consists of a (fixed) classical background $g_{\mu\nu}^{(0)}$ ("stage") with small quantum fluctuations $h_{\mu\nu}$ ("actors"). The "observable" (to be tested by a classical device) would be the effective matrix $g_{\mu\nu}^{(\text{eff})} = g_{\mu\nu}^{(0)} + < h_{\mu\nu} >$. Starting computations from the action (2) one finds that an ever increasing number of counter-terms is necessary. They are different from the terms in the Lagrangian $\mathcal{L} = \sqrt{-g}\, R / (16\pi G_N)$ in (2). This is the reason why QGR is called (perturbatively) "nonrenormalizable" [5]. Still, at energies $E \ll (G_N)^{-1/2}$, i.e. much below the Planck mass scale $m_{\text{Pl}} \propto (G_N)^{-1/2}$, such calculations can be meaningful in the sense of an "effective low energy field theory" [19], irrespective of the fact that (perhaps by embedding gravity into string theory) by inclusion of further fields at higher energy scales (Planck scale), QGR may become renormalizable. Of course, such an approach even when it is modified by iterative inclusion of $< h_{\mu\nu} >$ into $g_{\mu\nu}^{(0)}$ etc. – which is technically quite hopeless – completely misses inherent background independent effects, i.e. effects when $g_{\mu\nu}^{(0)} = 0$.

One could think also of applying nonperturbative methods developed in numerical lattice calculations for QCD. However, there are problems to define the Euclidean path integral for that, because the Euclidean action is not bounded from below (as it is the case in QCD) [20].

The quantization of gravity which – at least in principle – avoids background dependence is based upon the ADM approach to the Dirac quantization of the Hamiltonian [21]. Space-time is foliated by a sequence of three dimensional space-like manifolds $\sum_3$ upon which the variables $g_{ij} = q_{ij}$ and associated canonical momenta π_{ij} live. The constraints associated to the further variables lapse (N_0) and shift (N_i) in the Hamiltonian density

$$\mathcal{H} = N_0\, H^0\,(q,\pi) + N_i\, H^i\,(q,\pi) \tag{9}$$

are primary ones. The Poisson brackets of the secondary constraints H^μ closes. H^i generates diffeomorphisms on $\sum_3$. In the quantum version of (9) the solutions of the Wheeler-deWitt equation involving the Hamiltonian constraint

$$\int\limits_{\Sigma_3} H^0\left(q, \frac{\delta}{i\delta q}\right) \mid \psi >= 0 \tag{10}$$

formally would correspond to a nonperturbative QGR. Apart from the fact that it is extremely difficult to find a general solution to (10) there are

several basic problems with a quantum theory based upon that equation (e.g. no Hilbert space $\mid \psi >$ can be constructed, no preferred time foliation exists with ensuing inequivalent quantum evolutions [22], problems with usual "quantum causality" exist, the "axiom" that fields should commute at space like distances does not hold etc.). A restriction to a finite number of degrees of freedom ("mini superspace") [23] or infinite number of degrees of freedom (but still less than the original theory – so-called "midi superspace") [24] has been found to miss essential features.

As all physical states $\mid \psi >$ must be annihilated by the constraints H^μ, a naive Schrödinger equation involving the Hamiltonian constraint H^0,

$$i\hbar \frac{\partial \mid \psi >}{\partial t} = H^0 \mid \psi >= 0, \tag{11}$$

cannot contain a time variable ("problem of time"). A kind of Schrödinger equation can be produced by the definition of a "time-function" $T(q, \pi, x)$, at the price of an even more complicated formalism [25] with quite ambiguous results – and the problem, how to connect those with "genuine" observables. All these problems are aggravated, when one tries to first eliminate constraints by solving them explicitly before quantization. In this way, clearly part of the quantum fluctuations are eliminated from the start. As a consequence different quantum theories, constructed in this way, are not equivalent.

The "new" gravities (loop quantum gravity, spin foam models) reformulate the quantum theory of space-time by the introduction of novel variables, based upon the concept of Wilson loops (7) applied to the gauge-field (4). The operator

$$U(s_1, s_2) = \text{Tr}\, P\, \exp\left(i \int_{s_1}^{s_2} ds \frac{dx^i}{ds}\, A_i\right) \tag{12}$$

defines a holonomy. It is generalized by inserting further invariant operators at intermediate points between s_1 and s_2 . From such holonomies a spin network can be created which represents spacetime (in the path integral it is dubbed "spin foam").

These approaches claim several successes [2]. Introducing as a basis diffeomorphism equivalence classes of "labeled graphs" a finite Hilbert space can be constructed and some solutions of the Wheeler-deWitt equation (10) have been obtained. The methods introduce a "natural" coarse graining of space-time which implies a UV cutoff. "Small" gravity around certain states leads in those cases to corresponding linearized Einstein gravity.

However, despite of very active research in this field a number of very serious open questions persists: The Hamiltonian constructed from spin networks does not lead to massless excitations (gravitons) in the classical limit. The Barbero-Immirzi parameter γ has to be fixed by the requirement

of a "correct" Bekenstein-Hawking entropy for the Black Hole. The most severe problem, however, is the one of observables. By some researchers in this field it has been claimed that by "proper gauge fixing" (!) area and volume can be obtained as quantized "observables", which is a contradiction in itself from the point of view of QFT. We must emphasize too that also in an inherently UV regularized theory (finite) renormalization remains an issue to be dealt with properly. Also the fate of S-matrix elements, which play such a central role as the proper observables in QFT, is completely unclear in these setups.

Embedding QGR into (super-)string theory [26] does not remove the key problems related to the dual role of the metric. Gravity may well be a string excitation in a string/brane world of 10-11 dimensions, possibly a finite theory of everything. Nevertheless, at low energies Einstein gravity (eventually plus an antisymmetric B-field) remains the theory for which computations must be performed.[4] Unfortunately, the proper choice (let alone the derivation) of a string vacuum in our d=4 space-time is an unsolved problem.

Many other approaches exist, including noncommutative geometry, twistors, causal sets, 3d approaches, dynamical triangulations, Regge calculus etc., each of which has certain attractive features and difficulties (cf. e.g. [2] and refs. therein).

To us all these "new" approaches appear as – very ingenious – attempts to bypass the technical problems of directly applying standard QFT to gravity – without a comprehensive solution of the main problems of QGR being in sight. Thus the main points of a "minimal" QFT for gravity should be based upon "proven concepts" of QFT with a point of departure characterizing QGR as follows:

(a) QGR is an "effective" low energy theory and therefore need not be renormalizable to all orders.

(b) QGR is based upon classical Einstein (-dS) gravity with usual variables (metric or Cartan variables).

(c) At least the quantization of geometry must be performed in a background independent (nonperturbative) way.

(d) Absolutely "safe" quantum observables are only the S-matrix elements of QFT $< f \mid S \mid i >$, where initial state $\mid i >$ and final state $< f \mid$ are defined only when those states are realized as Fock states of particles in a (at least approximate) flat space environment. In certain cases it

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[4]It should be noted that the now widely confirmed astronomical observations of a positive cosmological constant [27] (if it is a constant and not a "quintessence" field in a theory of type [11]) precludes immediate application of supersymmetry (supergravity) in string theory, because only AdS space is compatible with supergravity [28].

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is permissible to employ a semi-classical approach: expectation values of quantum corrections may be added to classical geometric variables, and a classical computation is then based on the effective variables, obtained in this way.

Clearly item (d) by construction excludes any application to quantum cosmology, where $| i >$ would be the (probably nonexistent) infinite past before the Big Bang.

Obviously the most difficult issue is (c). We describe in the following section how gravity models in d=2 (e.g. spherically reduced gravity) permit a solution of just that crucial point, leading to novel results.

4 "Minimal" QGR in 1+1 dimensions

4.1 Classical theory: first order formulation

In the 1990s the interest in dilaton gravity in d=2 was rekindled by results from string theory [29], but it existed as a field on its own more or less since the 1980s [30]. For a review on dilaton gravity ref. [6] may be consulted. For sake of self-containment the study of dilaton gravity will be motivated briefly from a purely geometrical point of view.

The notation of ref. [6] is used: e^a is the zweibein one-form, $\epsilon = e^+ \wedge e^-$ is the volume two-form. The one-form ω represents the spin-connection $\omega^a{}_b = \varepsilon^a{}_b \omega$ with the totally antisymmetric Levi-Cività symbol ε_{ab} ($\varepsilon_{01} = +1$). With the flat metric η_{ab} in light-cone coordinates ($\eta_{+-} = 1 = \eta_{-+}$, $\eta_{++} = 0 = \eta_{--}$) the torsion 2-form reads $T^\pm = (d \pm \omega) \wedge e^\pm$. The curvature 2-form $R^a{}_b$ can be presented by the 2-form R defined by $R^a{}_b = \varepsilon^a{}_b R$, $R = d \wedge \omega$. Signs and factors of the Hodge-$*$ operation are defined by $*\epsilon = 1$.

Since the Hilbert action $\int_{\mathcal{M}_2} R \propto (1-g)$ yields just the Euler number for a surface with genus g one has to generalize it appropriately. The simplest idea is to introduce a Lagrange multiplier for curvature, X, also known as "dilaton field", and an arbitrary potential thereof, $V(X)$, in the action $\int_{\mathcal{M}_2} (XR + \epsilon V(X))$. In particular, for $V \propto X$ the Jackiw-Teitelboim model emerges [30]. Having introduced curvature it is natural to consider torsion as well. By analogy the first order gravity action [31]

$$L^{(1)} = \int_{\mathcal{M}_2} (X_a T^a + X R + \epsilon \mathcal{V}(X^a X_a, X)) \tag{13}$$

can be motivated where X_a are the Lagrange multipliers for torsion. It encompasses essentially all known dilaton theories in 2d, also known as Generalized Dilaton Theories (GDT). Spherically reduced gravity (SRG) from d=4 corresponds to $\mathcal{V} = -X^+ X^-/(2X) - \text{const.}$

Without matter there are no physical propagating degrees of freedom, which is advantageous mathematically but not very attractive from a physical point of view. Thus, in order to describe scattering processes matter

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has to be added. The simplest way is to consider a massless Klein-Gordon field ϕ,

$$L^{(m)} = \frac{1}{2} \int_{\mathcal{M}_2} F(X) \, d\phi \wedge *d\phi \,, \tag{14}$$

with a coupling function $F(X)$ depending on the dilaton (for dimensionally reduced theories typically $F \propto X$ holds).

4.2 Quantum theory: Virtual Black Holes

It turned out that even in the presence of matter an exact path integration of all geometric quantities is possible for all GDTs, proceeding along well established paths of QFT[5] [32].

The effective theory obtained in this way solely depends on the matter fields in which it is nonlocal and non-polynomial. Already at the level of the (nonlocal) vertices of matter fields, to be used in a systematic perturbative expansion in terms of Newton's constant, a highly nontrivial and physically intriguing phenomenon can be observed, namely the so-called "virtual black hole" (VBH). This notion originally has been introduced by S. Hawking [33], but in our recent approach the VBH for SRG emerges naturally in Minkowski signature space-time, without the necessity of additional *ad hoc* assumptions.

For non-minimally coupled scalars the lowest order S-matrix indeed exhibited interesting features: forward scattering poles, monomial scaling with energy, CPT invariance, and pseudo-self-similarity in its kinematic sector [34].

It was possible to reconstruct geometry self-consistently from a (perturbative or, if available, exact) solution of the effective theory. For the simplest case of four-point tree-graph scattering the corresponding Carter-Penrose (CP) diagram is presented in Fig. 1. It is non-local in the sense that it depends not only on one set of coordinates but on two. This was a consequence of integrating out geometry non-perturbatively. For each choice of y (one of the two sets of coordinates) it is possible to draw an ordinary CP-diagram. The non-trivial part of our effective geometry (i.e. the VBH) is concentrated on the light-like cut. For SRG the ensuing line-element has Sachs-Bondi form

$$(ds)^2 = 2drdu + \left(1 - \frac{2m(u,r)}{r} - a(u,r)r + d(u,r)\right)(du)^2 \,, \tag{15}$$

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[5]We mention just a few technical details: no ordering ambiguities arise, the (nilpotent) BRST charge is essentially the same as for Yang-Mills theory (despite of the appearance of nonlinearities in the algebra of the first class secondary constraints), the gauge fixing fermion is chosen such that "temporal" gauge is obtained, the Faddeev-Popov determinant cancels after integrating out the "unphysical" sector, and "natural" boundary conditions cannot be imposed on the fields, so one has to be careful with the proper treatment of the boundary.
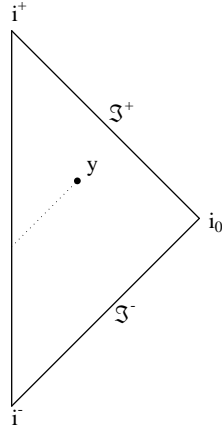
Figure 1: CP diagram of the VBH

with m, a and d being localized[6] on the cut $u = u_0$ with compact support $r < r_0$. These quantities depend on the second set of coordinates u_0, r_0.

One should not take the effective geometry at face value – this would be like over-interpreting the role of virtual particles in a loop diagram. It is a nonlocal entity and one still has to "sum" (read: integrate) over all possible geometries of this type in order to obtain the nonlocal vertices and the scattering amplitude. Nonetheless, the simplicity of this geometry and the fact that all possible configurations are summed over are nice features of this picture. Moreover, all VBH geometries coincide asymptotically and differ only very little from each other in the asymptotic region. This observation allows for the following interpretation: the boundaries of the diagram, $\mathscr{I}^\pm$ and i^0, behave in a classical way[7] (thus enabling one to construct an ordinary Fock space like in fixed background QFT), but the more one zooms into the geometry the less classical it becomes. The situation seems to be quite contrary to Kuchař's proposal of geometrodynamics[8] of BHs: while we have fixed boundary conditions for the target space coordinates (and hence a fixed ADM mass) but a "smeared geometry" (in the sense that a continuous spectrum of asymptotically equivalent VBHs contributes to the S-matrix), Kuchař encountered a "smeared mass" (obeying a Schrödinger equation) but an otherwise fixed geometry [35].

Qualitatively it is clear what has to be done in order to obtain the S-

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[6]The localization of "mass" and "Rindler acceleration" on a light-like cut is not an artifact of an accidental gauge choice, but has a physical interpretation in terms of the Ricci-scalar. Certain parallels to Hawking's Euclidean VBHs can be observed, but also essential differences. The main one is our Minkowski signature which we deem to be a positive feature.

[7]Clearly the imposed boundary conditions play a crucial role in this context. They produce effectively a fixed background, but only at the boundary.

[8]This approach considers only the matterless case and thus a full comparison to our results is not possible.

matrix[9]: Take all possible VBHs of Fig. 1 and sum them coherently with proper weight factors and suitably attached external legs of scalar fields. This had been done quantitatively in a straightforward but rather lengthy calculation for gravitational scattering of s-waves in the framework of SRG, the result of which yielded the lowest order tree-graph S-matrix for ingoing modes with momenta q, q' and outgoing ones k, k',

$$T(q, q'; k, k') = -\frac{i\kappa\delta\left(k + k' - q - q'\right)}{2(4\pi)^4|kk'qq'|^{3/2}} E^3\tilde{T},$$ (16)

with the total energy $E = q + q'$, $\kappa = 8\pi G_N$,

$$\tilde{T}(q, q'; k, k') := \frac{1}{E^3}\left[\Pi\ln\frac{\Pi^2}{E^6} + \frac{1}{\Pi}\sum_{p\in\{k,k',q,q'\}}\right.$$
$$\left. p^2\ln\frac{p^2}{E^2}\left(3kk'qq' - \frac{1}{2}\sum_{r\neq p}\sum_{s\neq r,p}\left(r^2s^2\right)\right)\right],$$ (17)

and the momentum transfer function $\Pi = (k + k')(k - q)(k' - q)$. The interesting part of the scattering amplitude is encoded in the scale independent factor $\tilde{T}$. The forward scattering poles occurring for $\Pi = 0$ should be noted.

It is possible to generalize the VBH phenomenon to arbitrary GDTs with matter as well as most of its properties (for instance, the CP-diagram, CPT invariance and the role played in the S-matrix) [37].

4.3   New results and outlook

Recently quantum corrections to the specific heat of the dilaton BH have been calculated by applying the quantization method discussed above [38]. The result is $C_s := dM/dT = 96\pi^2 M^2/\lambda^2$, where λ is the scale parameter of the theory. Thus, in that particular case quantum corrections lead to a stabilization of the system. The mass of the BH is found to be decreasing according to

$$M(u) \approx M_0 - \frac{\pi}{6}(T_H^0)^2(u - u_0) - \frac{\lambda}{24\pi}\ln\frac{M(u)}{M_0} + \mathcal{O}\left(\frac{\lambda}{M(u)}\right).$$ (18)

The first term is the ADM mass, the second term corresponds to a linear decrease due to the (in leading order) constant Hawking flux and the third term provides the first nontrivial correction.

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[9]The idea that BHs must be considered in the S-matrix together with elementary matter fields has been put forward some time ago [36]. The approach [34] reviewed here, for the first time allowed to derive (rather than to conjecture) the appearance of the BH states in the quantum scattering matrix of gravity.

Applying simple thermodynamical methods[10] ($dS = C_s dT/T$) and exploiting the quantum corrected mass/temperature relation $T/T_0 = 1 - \lambda/(48\pi M)$ it is possible to calculate also entropy corrections:

$$S = S_0 - \frac{1}{24}\ln S_0 + \mathcal{O}(1)\,, \quad S_0 := \frac{2\pi M}{\lambda} = 2\pi X|_{\text{horizon}} \qquad (19)$$

The logarithmic behavior is in qualitative agreement with the one found in the literature by various methods [40]; the prefactor $1/24$ coincides with [41].

An extension of the results obtained in the first order formulation to dilaton supergravity is straightforward in principle but somewhat tedious in detail. It permitted, among other results, to obtain for the first time a full solution of dilaton supergravity [42].

All these exciting applications indicate that the strict application of standard QFT concepts to gravity (at least in d=2 or in models dimensionally reduced to d=2) shows great promise.

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